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On type II strings in exact superconformal non-constant RR backgrounds

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Abstract: An explicitly exact superconformal description is provided to some classes of type II string theories in non constant RR backgrounds. This is done by applying the manifest (2,2) approach of Berkovits and Maldacena to type II strings and by studying the condition of exact conformal invariance of certain supersymmetric backgrounds. We find a new set of exact type-IIA strings with non constant RR 2-form and 4-form curvatures and for type IIB with non constant 3-form curvature.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background, Conformal Field Models in String Theory.

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Contents

1. Introduction

The study of string theory in curved backgrounds is crucial to understand which are its truly allowed vacua and to get new insights about its phase structure. Generally, the conformal invariance of a background — imposed mainly because of our actual inability in quantizing the non critical string σ-model coupled with the world-sheet Liouville field — is analyzed order by order in α' and there are very few handles on exactly decoupled string theories.

It appeared recently an interesting class of pp-wave solutions of type IIB [1], generalizing the one studied in [2], with non constant RR fluxes. These string theories have been shown to admit a superconformal formulation in [3] and have been shown to be exact (finite) in [3, 4]. These solutions have been studied in the SU(4) × U(1) formalism [5, 6]. Within this framework the type-IIB GS action on a flat background is written in terms of four complex chiral (2,2) superfields $X^{+\bar{l}}$, where $l = 1, \ldots, 4$, as

$$S_0^{\mathrm{IIB}} = \frac{1}{2}\int d^2z d^4\kappa X^{+\bar{l}} X^{-l}\,, \tag{1.1}$$

where $D_- X^{+\bar{l}} = 0$, $\bar{D}_- X^{+\bar{l}} = 0$ and $D_+ X^{-l} = 0$, $\bar{D}_+ X^{-l} = 0$. Here $D_\pm$ and $\bar{D}_\pm$ are the usual superderivatives. The only subgroup of the original SO(8) symmetry which remains manifest is a SU(4) × U(1) one.

Upon improving and completing the theory with a proper set of free chiral and semi-chiral multiplets, the theory can be shown to admit an equivalent explicitly superconformal description. In particular, there are two kind of extensions to curved backgrounds with

RR fields, which preserve respectively (1,1) and (2,2) supersymmetry. These extensions are quite special since they keep the exact superconformal symmetry. This means that once the superconformal invariance is established at one loop, than it is implied also at higher loops just because of algebraic (in the superspace) reasons. All this is well explained in [3] and will not be reviewed here. Other papers related to these subjects and issues are [7]–[15].

A first aim of this letter is to study extensions of the above improved superconformal type-IIA theory which remain exact string theories in parallel with the analysis performed in [3] for the type-IIB case. Actually, a partial extension, linked by explicit T-duality to a restricted (2,2) preserving case, was already considered in [7]. Here we will give a notably wider set of possibilities. Moreover, we will find another class of exact type-IIB backgrounds by using a different manifest superconformal picture. This will turn out to be the natural picture to describe type-IIB strings in pp-wave backgrounds with non constant RR 3-form curvature.

2. Type-IIA exact strings in non constant RR 2 and 4-forms

2.1 The type-IIA $\mathrm{U}(3)\times\mathrm{U}(1)$ formalism

Because of the different chirality assignment between left and right moving fermions, as it was shown in [7, 8], the analogous type-IIA (2,2) σ-model has three chiral superfields $\Phi^{\bar{i}}$, where $i=1,2,3$, and a twisted chiral one that we call Σ. This is the type IIA counterpart of the above superfields formalism in which only a $\mathrm{U}(3)\times\mathrm{U}(1)$ symmetry is manifest because of the necessity of introducing an asymmetric chirality structure to the four supermultiplets. The type-IIA action is then given by

$$S_0^{\mathrm{IIA}}=\frac{1}{2}\int d^2zd^4\kappa\left\{-\Sigma\bar{\Sigma}+\Phi^i\Phi^{\bar{i}}\right\}. \tag{2.1}$$

The twisted chiral superfields satisfy by definition $D_-\Sigma=0$, $\bar{D}_+\Sigma=0$ and $\bar{D}_-\bar{\Sigma}=0$, $D_+\bar{\Sigma}=0$.

The action (2.1) can be obtained from (1.1) by T-dualizing along one of the eight transverse directions, and vice versa. More explicitly[1] let X and $\bar{X}$ be chiral and anti-chiral super fields respectively, B a general real superfield, Σ and $\bar{\Sigma}$ twisted chiral and anti-chiral super fields. Let us consider the action

$$S_0=\int d^2zd^4\kappa\left(\frac{1}{4}B^2-\frac{1}{2}B(\Sigma+\bar{\Sigma})\right). \tag{2.2}$$

We can consider the theory defined by the action S_0 in two equivalent ways.

We can complete the squares

$$S_0=\int d^2zd^4\kappa\left(\frac{1}{4}(B-\Sigma-\bar{\Sigma})^2-\frac{1}{4}(\Sigma+\bar{\Sigma})^2\right)$$

and integrate over the decoupled field $B'=B-\Sigma-\bar{\Sigma}$ by leaving an effective action

$$\tilde{S}_0(\Sigma)=\int d^2zd^4\kappa\frac{-1}{4}(\Sigma+\bar{\Sigma})^2=-\frac{1}{2}\int d^2zd^4\kappa\Sigma\bar{\Sigma}\,.$$

[1] We follow and shortly review the approach of [16] and [17].

Otherwise, we can integrate over Σ and $\bar{\Sigma}$ which induce the constrains

$$\bar{D}_+ D_- B = 0 \qquad \text{and} \qquad D_+ \bar{D}_- B = 0 \,.$$

These constrains are solved by $B = X + \bar{X}$ for some chiral X (and anti-chiral $\bar{X}$). Recasting this in the remnant of S_0, we find the dual description

$$S_0(X) = \int d^2 z d^4 \kappa \frac{1}{4} \left(X + \bar{X} \right)^2 = \frac{1}{2} \int d^2 z d^4 \kappa X \bar{X} \,.$$

The same completing argument which applied to (1.1), can be invoked for (2.1) too. In detail, following [3], we add four chiral fermionic multiplets $\Theta^{\pm}$ and $\bar{\Theta}^{\pm}$ and four semichiral fermionic multiplets $W^{\pm}$ and $\bar{W}^{\pm}$ and [2] improve the action (2.1) by the addition of

$$S_{\theta w} = \frac{1}{2} \int d^2 z d^4 \kappa \left\{ W^+ \Theta^- + W^- \Theta^+ + \bar{W}^+ \bar{\Theta}^- + \bar{W}^- \bar{\Theta}^+ \right\} , \tag{2.3}$$

where $\bar{D}_{\pm} W^{\mp} = 0$, $D_{\pm} \bar{W}^{\mp} = 0$ are the conditions defining the semichiral super-fields and $D_{\pm} \Theta^{\mp} = 0$, $D_{\pm} \bar{\Theta}^{\mp} = 0$ $\bar{D}_{\pm} \Theta^{\mp} = 0$ $\bar{D}_{\pm} \bar{\Theta}^{\mp} = 0$ are the conditions defining the chiral super-fields. Since the central charges of chiral and twisted chiral multiplets are of course equals, the improved theory is manifestly superconformal in the type-IIA case too.

Notice that the improved action $S_0^{IIA} + S_{\theta w}$ is invariant under the following supersymmetry transformations

$$\begin{aligned}
\delta \Theta^{\pm} &= \epsilon^{\pm} , & \delta \bar{\Theta}^{\pm} &= \bar{\epsilon}^{\pm} \\
\delta \Phi^{i} &= \epsilon^{+i} \Theta^- + \bar{\epsilon}^{+i} \bar{\Theta}^- , & \delta \Phi^{\bar{i}} &= \epsilon^{-\bar{i}} \Theta^+ + \bar{\epsilon}^{-\bar{i}} \bar{\Theta}^+ \\
\delta \Sigma &= 0 , & \delta \bar{\Sigma} &= 0 \\
\delta W^+ &= -\epsilon^{+i} \Phi^{\bar{i}} , & \delta W^- &= -\epsilon^{-\bar{i}} \Phi^{i} \\
\delta \bar{W}^+ &= -\bar{\epsilon}^{+i} \Phi^{\bar{i}} , & \delta \bar{W}^- &= -\bar{\epsilon}^{-\bar{i}} \Phi^{i}
\end{aligned} \tag{2.4}$$

as well as under the fermionic symmetries

$$\begin{aligned}
\delta' \Sigma &= \bar{D}_+ \bar{W}^+ \bar{\zeta} + D_- W^- \eta , & \delta' \bar{\Sigma} &= D_+ W^+ \zeta + \bar{D}_- \bar{W}^- \bar{\eta} \\
\delta' \Theta^+ &= D_- \bar{\Sigma} \eta , & \delta' \Theta^- &= D_+ \Sigma \zeta \\
\delta' \bar{\Theta}^+ &= \bar{D}_- \Sigma \bar{\eta} , & \delta' \bar{\Theta}^- &= \bar{D}_+ \bar{\Sigma} \bar{\zeta} \\
\delta' W^{\pm} &= 0 , & \delta' \bar{W}^{\pm} &= 0 , \qquad \delta' \Phi^{i} = 0 , \qquad \delta' \Phi^{\bar{i}} = 0
\end{aligned} \tag{2.5}$$

which have no direct counterpart in the manifest U(4) type-IIB case (their type-IIB counterpart will appear in the next section).

2.2 pp-waves in type IIA

Due to the light-cone interpretation of the ΘW sector, looking for pp-wave backgrounds corresponds to add to the free improved action $S_0^{IIA} + S_{\theta w}$ a further interacting term which does not modify the Θ's equation of motion. Equivalently, requiring x^- independence of the background, implies the independence of any further interaction term upon the W's.

[2]In the following we will refer to this sector of the theory as to the ΘW sector.

JHEP01(2003)065

The general type-IIA supergravity background we refer to is given by the metric[3]

$$ds_{10}^2 = -2dx^+dx^- + H(dx^+)^2 + d\phi^i d\phi^{\bar{i}} + d\sigma d\bar{\sigma} \tag{2.6}$$

and the RR-field curvatures

$$F_{(4)} = dx^+ \wedge \omega_{(3)} \qquad F_{(2)} = dx^+ \wedge \omega_{(1)} \,. \tag{2.7}$$

This class of backgrounds satisfies the supergravity equations of motion if the $\omega_{(i-1)}$ are harmonic forms in the transverse eight dimensional space and if the pp-wave factor H satisfies the equation

$$\left(\partial_i \partial_{\bar{i}} + \partial_\sigma \partial_{\bar{\sigma}}\right) H \propto |\omega_{(3)}|^2 + |\omega_{(1)}|^2 \tag{2.8}$$

which is the only non trivial equation for the metric, namely $R_{++} + T_{++} = 0$.

Therefore, we consider the most general classically (2,2) superconformal action for pp-waves that is

$$S_{pp} = S_0^{IIA} + S_{\theta w} + \int d^2 z d^4 \kappa U \left(\Phi^i, \Sigma, \Phi^{\bar{i}}, \bar{\Sigma}, \Theta^\pm, \bar{\Theta}^\pm\right), \tag{2.9}$$

where U is a general bosonic superfield function. Restricting to polynomial dependence on the superfields and requiring that U effectively generates RR vertices, we consider a general bosonic superfield at most quadratic in the Θs and containing an odd number of barred and unbarred fermions.

We would like now to specify few cases preserving some of the supersymmetries (2.4) above and which are exactly superconformal.

(2,2) supersymmetry A first class is the one with all (2,2) supersymmetry, i.e. the $\epsilon^\pm$ and the $\bar{\epsilon}^\pm$, where one takes

$$U\left(\Phi^i, \Sigma, \Phi^{\bar{i}}, \bar{\Sigma}, \Theta^\pm, \bar{\Theta}^\pm\right) = \Theta^+\bar{\Theta}^+\mathcal{F}(\Phi^i) + \Theta^-\bar{\Theta}^-\bar{\mathcal{F}}(\Phi^{\bar{i}}) + \Theta^+\bar{\Theta}^-\mathcal{E}(\Sigma) + \Theta^-\bar{\Theta}^+\bar{\mathcal{E}}(\bar{\Sigma}),$$

where $\mathcal{F}$ and $\mathcal{E}$ are holomorphic functions of the Φ^i's and Σ respectively.

Notice that the full action is still invariant under the supersymmetries $\epsilon^\pm$ and $\bar{\epsilon}^\pm$ if we modify the $\delta W = 0$ rule (with respect to these variations) to

$$\begin{aligned} \delta W^- &= \bar{\epsilon}^+\mathcal{F} + \bar{\epsilon}^-\mathcal{E} \qquad & \delta W^+ &= \bar{\epsilon}^-\bar{\mathcal{F}} + \bar{\epsilon}^+\bar{\mathcal{E}} \\ \delta \bar{W}^- &= -\epsilon^+\mathcal{F} - \epsilon^-\bar{\mathcal{E}} \qquad & \delta \bar{W}^+ &= -\epsilon^-\bar{\mathcal{F}} - \epsilon^+\mathcal{E} \,. \end{aligned} \tag{2.10}$$

This is consistent with the semichiral properties of the Ws.

It is also possible to modify the fermionic symmetry (2.5) in such a way that the action remains invariant. This happens if $\mathcal{F} = 0$ and, in this case, one just has to modify the $\delta' W = 0$ rule in (2.5) to

$$\begin{aligned} \delta' W^- &= -\mathcal{E}\bar{D}_+\bar{\Sigma}\bar{\zeta} \qquad & \delta' W^+ &= -\bar{\mathcal{E}}\bar{D}_-\Sigma\bar{\eta} \\ \delta' \bar{W}^- &= -\bar{\mathcal{E}}D_+\Sigma\zeta \qquad & \delta' \bar{W}^+ &= -\mathcal{E}D_-\bar{\Sigma}\eta \,. \end{aligned} \tag{2.11}$$

[3] We already distinguish the two directions along which the scalar component of the twisted multiplet elongates the string and the six direction relative to the chiral triplet superfield scalar components.

Notice that this peculiar extension shows the nature of this further fermionic symmetry which acts transversely with respect to the chiral supersymmetry structure. Notice, in particular, that if the chiral superpotential $\mathcal{F}$ is left non vanishing, then one finds in the δ' variation of the action terms like $\int \theta^- \bar{\mathcal{F}} \bar{D}_+ \bar{\Sigma} \bar{\zeta}$ which can not be reabsorbed nor with a consistent shift in $\delta'\Sigma$ nor in $\delta' W^+$, where consistency is understood with respect to the chiral/anti-chiral constrains on the superfields.

The same argument as the one given in [3], leads to exact conformal invariance of these models. Notice that, upon on-shell reduction of the ΘW sector, the (2,2) σ-model action is

$$\frac{1}{2}\int d^2z d^4\kappa \left\{-\Sigma\bar{\Sigma} + \Phi^i \Phi^{\bar{i}}\right\} + \frac{1}{2}\int d^2z d\kappa^- d\bar{\kappa}^- \mathcal{F}(\Phi^i)|_{\kappa^+=0=\bar{\kappa}^+} + \\ + \int d^2z d\kappa^- d\bar{\kappa}^+ \mathcal{E}(\Sigma)|_{\kappa^+=0=\bar{\kappa}^-} + \text{h.c.} \tag{2.12}$$

This corresponds to the following type-IIA background. The pp-wave factor H in (2.6) is

$$H = |\partial_i \mathcal{F}|^2 + |\partial_\sigma \mathcal{E}|^2$$

and the harmonic forms are

$$\omega_{(1)} = \sqrt{2}\text{Re}\left\{\partial_\sigma^2 \mathcal{E} d\sigma\right\}$$

and

$$\omega_{(3)} = \text{Re}\left\{\epsilon_{\bar{i}\bar{j}}^{\ \ l}\partial_l \partial_k \mathcal{F} d\phi^{\bar{i}} \wedge d\phi^{\bar{j}} \wedge d\phi^k\right\}$$

which solve equation (2.8).

Notice that if in particular one restricts to the case $\mathcal{E} = 0$, the background already considered in [7] is recovered as a sub-case.

(1,1) extra supersymmetry. A second class is the one with extra $(1,1)$ supersymmetry $\epsilon^+ = \epsilon^-$ and $\bar{\epsilon}^+ = \bar{\epsilon}^-$ where one takes

$$U\left(\Phi^i, \Sigma, \Phi^{\bar{i}}, \bar{\Sigma}, \Theta^\pm, \bar{\Theta}^\pm\right) = \left(\Theta^+ - \Theta^-\right)\left(\bar{\Theta}^+ - \bar{\Theta}^-\right) V\left(\Phi^i, \Sigma, \Phi^{\bar{i}}, \bar{\Sigma}\right)$$

and the condition of exact superconformal invariance takes the form of

$$\left(\partial_\Sigma \partial_{\bar{\Sigma}} + \partial_i \partial_{\bar{i}}\right) V = 0 \tag{2.13}$$

that is V is an harmonic function in the transverse eight dimensional space. Notice that the sign in front of the $\partial_\Sigma \partial_{\bar{\Sigma}}$ in equation (2.13) combines the minus sign of the super-propagator of twisted chiral fields and the minus sign in the mixed $\Theta^\pm \bar{\Theta}^\mp$ on shell contractions. This crucial sign (for comparison with supergravity) can also be checked by examining the highest component of the stress-energy tensor and by requiring the absence of double pole with V. This condition reproduces directly the harmonic equation (2.13).

This corresponds to the following type-IIA background. The pp-wave factor H in (2.6) is

$$H = |\partial_\sigma V|^2 + |\partial_i V|^2$$

and the harmonic forms are

$$\omega_{(1)} = d\left[\partial_\sigma V\right] + \text{h.c.}$$

and

$$\omega_{(3)} = d\left[\partial_i V \epsilon^i{}_{\bar{k}\bar{l}} d\phi^{\bar{k}} \wedge d\phi^{\bar{l}} + \text{h.c.}\right],$$

where d is the usual differential of forms in the transverse eight dimensional space. Notice that if V is independent on the twisted chiral superfields, then the above solution could have been obtained by T-duality from an analogous type-IIB case with $(1,1)$ supersymmetry.

Also for the more general type-IIA background geometries that we considered here it is possible to formulate a matrix string model by the appropriate non-abelian gauging of the σ-model, as in [7, 8, 18].

3. Exact backgrounds for type IIB with RR 3-form

3.1 The U(2) × U(2) formalism for type IIB

As it appears clearly from the introduction, there exists different possible $(2,2)$ manifest superconformal formalisms for type II string theories on flat background. These pictures can be obtained by formally T-dualizing along the way we exposed in the introduction.

As far as the type-IIA case is concerned, the other possible picture, that is with one chiral and three twisted chiral superfields, is equivalent to the one with three chiral and one twisted chiral superfields just by a relabeling of the superspace variables (who is chiral and who is antichiral makes sense only relatively). Therefore, the type-IIA picture we studied so far is exhaustive of the inequivalent cases also after the addition of the RR-background fields.

As far as the type-IIB case is concerned, we can represent type IIB on a flat background in two equivalent ways. One is the U(4) manifest picture with four chiral multiplets. Another one is with two chiral and two twisted chiral multiplets with action

$$S' = \frac{1}{2}\int d^2z d^4\kappa \left(\Phi^a\Phi^{\bar{a}} - \Sigma^{a'}\Sigma^{\bar{a}'}\right) \tag{3.1}$$

which exhibits a manifest U(2) × U(2) rotational symmetry (here $a, a' = 1, 2$). The picture (3.1) is equivalent to the U(4) manifest one (by two T-dualities) on the flat background.

Obviously, a picture with four twisted chiral superfields and no chiral one is equivalent to the one with four chiral superfields just by a relabeling of the superspace variables.

After the introduction of the RR curvature field backgrounds, the picture (3.1) is no longer generically equivalent to the manifest U(4) and allows the description of other exact string theories on curved backgrounds of pp-wave type.

Possible inequivalent RR-backgrounds for type IIB which preserve some supersymmetry and are exactly superconformal can be again obtained by the method of adding a vertex quadratic in the Θs which does not generate conformal anomalies because of algebraic reasons.

Before studying these backgrounds, we notice that the ΘW improved type-IIB action $S' + S_{\theta w}$ is invariant under the following supersymmetries which act transversely with respect to the chiral structure of the σ-model picture. These are

$$\begin{aligned} \delta\Theta^{\pm} &= \epsilon^{\pm}\,, & \delta\bar{\Theta}^{\pm} &= \bar{\epsilon}^{\pm} \\ \delta\Phi^{a} &= \epsilon^{a}\Theta^{-} + \bar{\epsilon}^{a}\bar{\Theta}^{-}\,, & \delta\Phi^{\bar{a}} &= \epsilon^{\bar{a}}\Theta^{+} + \bar{\epsilon}^{\bar{a}}\bar{\Theta}^{+} \\ \delta\Sigma^{a'} &= 0\,, & \delta\Sigma^{\bar{a}'} &= 0 \\ \delta W^{+} &= -\epsilon^{a}\phi^{\bar{a}}\,, & \delta W^{-} &= -\epsilon^{\bar{a}}\phi^{a} \\ \delta\bar{W}^{+} &= -\bar{\epsilon}^{a}\phi^{\bar{a}}\,, & \delta\bar{W}^{-} &= -\bar{\epsilon}^{\bar{a}}\phi^{a} \end{aligned} \tag{3.2}$$

as well as under the fermionic symmetries

$$\begin{aligned} \delta'\Sigma^{a'} &= \bar{D}_{+}\bar{W}^{+}\bar{\zeta}^{a'} + D_{-}W^{-}\eta^{a'}\,, & \delta'\Sigma^{\bar{a}'} &= D_{+}W^{+}\zeta^{\bar{a}'} + \bar{D}_{-}\bar{W}^{-}\bar{\eta}^{\bar{a}'} \\ \delta'\Theta^{+} &= D_{-}\Sigma^{\bar{a}'}\eta^{a'}\,, & \delta'\Theta^{-} &= D_{+}\Sigma^{a'}\zeta^{\bar{a}'} \\ \delta'\bar{\Theta}^{+} &= \bar{D}_{-}\Sigma^{a'}\bar{\eta}^{\bar{a}'}\,, & \delta'\bar{\Theta}^{-} &= \bar{D}_{+}\Sigma^{\bar{a}'}\bar{\zeta}^{a'} \\ \delta' W^{\pm} &= 0\,, & \delta'\bar{W}^{\pm} &= 0\,, \qquad \delta'\Phi^{a} = 0\,, \qquad \delta'\Phi^{\bar{a}} = 0\,. \end{aligned} \tag{3.3}$$

These are the analogous to eqs. (2.5) for type IIB in the picture with two chiral and two twisted chiral superfields.

3.2 Type-IIB pp-waves with RR 3-form backgrounds

Also the picture (3.1) allows a family of interaction vertices preserving some supersymmetries and manifest conformal invariance.

These parallel the (1,1) and (2,2) preserving cases given for type IIA in the previous section and for type IIB in [3]. The novelty in the case at hand is that the pp-waves naturally support 3-form RR field curvatures.

The background we consider now is

$$ds_{10}^{2} = -2dx^{+}dx^{-} + H(dx^{+})^{2} + d\phi^{a}d\phi^{\bar{a}} + d\sigma^{a'}d\bar{\sigma}^{\bar{a}'} \tag{3.4}$$

and the RR-field curvature

$$F_{(3)} = dx^{+}\wedge\omega_{(2)}\,. \tag{3.5}$$

This class of backgrounds satisfies the supergravity equations of motion if $\omega_{(2)}$ is harmonic in the transverse eight dimensional space and if the pp-wave factor H satisfies the equation

$$\left(\partial_{a}\partial_{\bar{a}} + \partial_{a'}\partial_{\bar{a}'}\right)H \propto |\omega_{(2)}|^{2} \tag{3.6}$$

which is the non trivial equation for the metric.

(1,1) preserving case. The *(1,1) preserving case* corresponds to the addition to the ΘW-improved action $S' + S_{\theta w}$ of the vertex

$$U\left(\Phi^{a},\Sigma^{a'},\Phi^{\bar{a}},\Sigma^{\bar{a}'},\Theta^{\pm},\bar{\Theta}^{\pm}\right) = \left(\Theta^{+}-\Theta^{-}\right)\left(\bar{\Theta}^{+}-\bar{\Theta}^{-}\right)V\left(\Phi^{a},\Sigma^{a'},\Phi^{\bar{a}},\Sigma^{\bar{a}'}\right)$$

and the condition of exact superconformal invariance takes the form of

$$\left(\partial_{a}\partial_{\bar{a}} + \partial_{a'}\partial_{\bar{a}'}\right)V = 0 \tag{3.7}$$

that is V again a harmonic function in the transverse eight dimensional space.

This corresponds to the following type-IIB background. The pp-wave factor H in (3.4) is

$$H = |\partial_a V|^2 + |\partial_{a'} V|^2$$

and the harmonic 2-form is

$$\omega_{(2)} = d\left[\partial_a V \epsilon^a_{\bar{b}} d\phi^{\bar{b}} + \partial_{a'} V \epsilon^{a'}_{\bar{b}'} d\phi^{\bar{b}'}\right] + \text{h.c.}\,,$$

where d is the form differentiation in the transverse eight dimensional space.

(2,2) preserving case. The *(2,2) preserving case* corresponds instead to the addition to the ΘW-improved action of the vertex

$$U\left(\Phi^a, \Sigma^{a'}, \Phi^{\bar{a}}, \Sigma^{\bar{a}'}, \Theta^\pm, \bar{\Theta}^\pm\right) = \Theta^+\bar{\Theta}^+\mathcal{F}(\Phi^a) + \Theta^-\bar{\Theta}^-\bar{\mathcal{F}}(\Phi^{\bar{a}}) + \Theta^+\bar{\Theta}^-\mathcal{E}(\Sigma^{a'}) + \\ + \Theta^-\bar{\Theta}^+\bar{\mathcal{E}}(\Sigma^{\bar{a}'})\,,$$

where $\mathcal{F}$ and $\mathcal{E}$ are holomorphic functions of the ϕ^a's and $\Sigma^{a'}$ respectively.

Notice that the full action is still invariant under the supersymmetries $\epsilon^\pm$ and $\bar{\epsilon}^\pm$ if we modify the $\delta W = 0$ rule (with respect to these variations) as for the type-IIA case to

$$\begin{aligned} \delta W^- &= \bar{\epsilon}^+\mathcal{F} + \bar{\epsilon}^-\mathcal{E} & \qquad \delta W^+ &= \bar{\epsilon}^-\bar{\mathcal{F}} + \bar{\epsilon}^+\bar{\mathcal{E}} \\ \delta\bar{W}^- &= -\epsilon^+\mathcal{F} - \epsilon^-\bar{\mathcal{E}} & \qquad \delta\bar{W}^+ &= -\epsilon^-\bar{\mathcal{F}} - \epsilon^+\mathcal{E}\,. \end{aligned} \tag{3.8}$$

Again this is consistent with the semichiral properties of the Ws.

It is also possible to modify the fermionic symmetry (3.3) in such a way that the action remains invariant. This happens if $\mathcal{F} = 0$ and, in this case, one just has to modify the $\delta' W = 0$ rule in (3.3) to

$$\begin{aligned} \delta' W^- &= -\mathcal{E}\bar{D}_+\Sigma^{\bar{a}'}\bar{\zeta}^{a'} & \qquad \delta' W^+ &= -\bar{\mathcal{E}}\bar{D}_-\Sigma^{a'}\bar{\eta}^{\bar{a}'} \\ \delta'\bar{W}^- &= -\bar{\mathcal{E}}D_+\Sigma^{a'}\zeta^{\bar{a}'} & \qquad \delta'\bar{W}^+ &= -\mathcal{E}D_-\Sigma^{\bar{a}'}\eta^{a'}\,. \end{aligned} \tag{3.9}$$

Notice that this peculiar extension shows again the nature of this further fermionic symmetry which acts transversely with respect to the chiral supersymmetry structure.

Let us notice that the construction performed in this section is complementary to the analysis of these backgrounds done with other methods by Russo and Tseytlin in [4].

4. Conclusions and open questions

In this letter we developed and generalized the approach first proposed by Berkovits and Maldacena in [3] to characterize exact string theories on pp-wave backgrounds sustained by non constant RR curvature forms. Our main two results are the generalization of the hybrid formalism to type-IIA string theory and the formulation of a complementary hybrid formalism for type IIB. By using these tools, we are able to describe new exact superconformal σ-models for string theories coupled to new pp-wave backgrounds sustained by different RR curvature forms.

JHEP01(2003)065

4.1 Other backgrounds

Another possible class of backgrounds which can be shown easily to be exact are the ones with constant B field in the eight dimensional transverse space.

That's because, as shown in [16], mixed terms in the quadratic free action involving chiral and twisted chiral correspond to constant B fields. Therefore, we can add for free to (2.1) the term [4]

$$\int d^2zd^4\kappa \left[B^1_i\phi^i + B^2_{\bar{i}}\phi^{\bar{i}}\right]\Sigma + \text{h.c.} \tag{4.1}$$

Since the σ-model action is still free, it is trivially superconformal exact. This corresponds to a flat ten dimensional background with a constant B field turned on in the eight directions transverse to the light-cone.

Yet another class of backgrounds which are exactly superconformal are the dilatonic ones studied in [10]. These can be adapted easily to the type-IIA string too.

A more general class of type-IIA backgrounds which it would be very nice to understand in the superfield formalism is that of pp-waves warped by an harmonic functions as

$$ds^2_{10} = A^{-3/4}\left[-2dx^+dx^- + H(dx^+)^2\right] + A^{1/4}\left[d\phi^i d\phi^{\bar{i}} + d\sigma d\bar{\sigma}\right] \tag{4.2}$$

with a dilaton and NSNS-field curvature

$$\Phi = -\frac{1}{2}\log A \qquad G_{(3)} = dA^{-1}\wedge dx^+ \wedge dx^- \tag{4.3}$$

and the RR-field curvatures

$$F_{(4)} = dx^+ \wedge \omega_{(3)} \qquad F_{(2)} = dx^+ \wedge \omega_{(1)}\,. \tag{4.4}$$

This class of backgrounds satisfies the supergravity equations of motion if the $\omega_{(i-1)}$ are harmonic forms in the transverse eight dimensional space, if the pp-wave factor H satisfies the equation

$$\left(\partial_i\partial_{\bar{i}} + \partial_\sigma\partial_{\bar{\sigma}}\right) H \propto |\omega_{(3)}|^2 + |\omega_{(1)}|^2 \tag{4.5}$$

and if the warp factor A is an harmonic function in the eight dimensional space.

These generalize the backgrounds studied in [19], naively reduce for $A = 1$ to the ones we studied in section 2 of this letter and are one of their most natural generalizations. A possibility for their implementation in the above formalism [20] maybe in a non trivial W dependence in the improved σ-model action in the style of [10] to reproduce the non constant warping factor and dilaton.

4.2 Non-local symmetries and RR-vertices

There is increasing evidence that the M-theory picture for string theory has to include also bosonic string. This has been discussed for example in [21], where ten dimensional superstrings on flat space-time, as well as type 0s, have been rebuilt from the critical bosonic 26 dimensional string theory in flat spacetime. On the other way around, the embedding of $N = i$ in $N = i+1$, for $i = 0, 1$, string theories was established in [22]. If it is

[4] There is an analogous one for (3.1) too.

true that all known string theories are one, it has to be true that for any consistent choice of the background, superstring has a purely bosonic string counterpart on some related dual background geometry.

It is therefore important to understand how this happens, or has chances to happen, on the exact backgrounds that we are considering. A first suggestion has been given in [12], where the $N = 0$ to $N = 2$ correspondence for certain integrable systems has been recalled from [23].

In summary, the correspondence is a duality between the chiral ring structure of a $N = 2$ theory, which is defined by its superpotential function, and the vertex algebra in the integrable $N = 0$ theories. Algebraically, the map is established at the points in the moduli space of couplings of $N = 0$ theories where a $\mathcal{U}_q\left(\hat{sl}(2)\right)$ affine quantum symmetry arises with $q = \pm i$. In this case, the affine quantum algebra structure reproduces directly the $N = 2$ superalgebra.[5]

Typically, to reproduce the kind of background we are considering here, namely with non compact transverse eight dimensional space — due to the fact that we require the existence of non constant holomorphic functions on such a space — one might consider generalized (non conformal) affine Toda theories.

The full string theory off shell symmetry, namely the conformal symmetry, might then be recovered via further affinization (as suggested in [12] for the simpler case), by generalizing the construction [24]. It remains a crucial issue to understand if the ΘW sector completition of the $N = 2$ theory is map to the affinization to the conformal theory of the $N = 0$ model counterpart or if this point is still problematic and requires a deeper analysis.

Notice that there is a further link between the quantum groups invariance and a particular feature of string theory on RR-background.

As it is well known, the quantum group symmetry which is typical of the massive integrable two dimensional field theories we are using to model string theories in RR-background geometries, extends non trivially the classical Colemann-Mandula theorem because it does not act on multiparticle states as if they were tensor products of one particle states. Technically, this happens because of the non triviality of the comultiplication as an Hopf algebra. Equivalently, from the quantum field theory point of view, the preserved charges are given by integrals of time components of currents which are not local with respect to the elementary fields creating one particle states. Therefore, the symmetry is of a non local type and requires to be realized in a theory with vertices which are non local with respect to the currents.

On the string theory side, we know that the equivalence of the GS formalism to the covariant RNS is non local, because of the bosonization involved while implementing the SO(8) triality transformations for the world sheet fermion fields. As a consequence, the RR-vertices are mapped to bilinears of the spin operators which are non local with respect to the world-sheet fields. This generates well known problems in forcing the RNS description for RR-backgrounds with the usual local σ-model techniques.

[5]It is very much tempting to consider the more general Drinfeld quantum double of the yangian $\mathcal{U}_q\left(\hat{\mathfrak{g}}\right)$ symmetry of massive WZNW models — i.e. such that the kinetic term coefficient is not matched to the level $\mathtt{k}$ which appears as prefactor of the WZ term — as the $N = 0$ starting point for a generalization.

JHEP01(2003)065

We believe that these manifestation of non-localities in the two dimensional quantum field theory algebras should be somehow linked and deeping this analysis could hopefully clarify the intricacies of the nonlocal change of variable [10] needed to link our approach to the covariant R-NS one.

In conclusion let us add still a couple of observations. The extension to non flat transverse eight-dimensional space geometries of the formalisms which has been developed here is possible. Moreover, it would be of extreme interest to have a link with the techniques of [25, 26] to describe string theories on Calabi-Yau's and geometric transitions in presence of RR-backgrounds at the microscopic σ-model level.

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